

# EE 435

## Lecture 4

### Fully Differential Single-Stage Amplifier Design


- General Differential Analysis
- 5T Op Amp from simple quarter circuit
- Biasing with CMFB circuit
- ⇒• Common-mode and differential-mode analysis
- ⇒• Common Mode Gain
- ⇒• Overall Transfer Characteristics

Design of 5T Op Amp

Slew Rate

Review from last lecture:  
Where we are at:

# Basic Op Amp Design

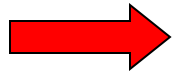
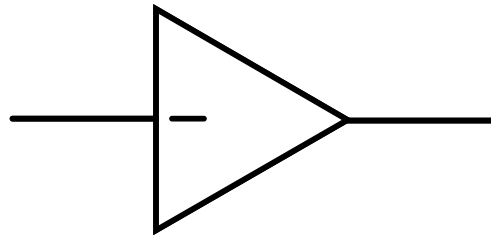
- Fundamental Amplifier Design Issues
-  • Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Two-Stage Op Amp
- Other Basic Gain Enhancement Approaches

Review from last lecture:

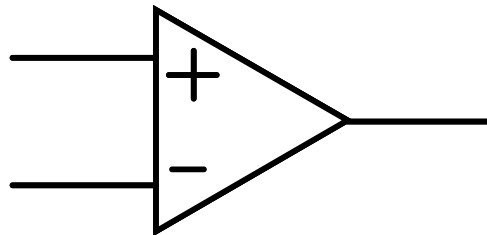
Where we are at:

# Single-Stage Low-Gain Op Amps

- Single-ended input



- Differential Input



**(Symbol does not distinguish between different amplifier types)**

Review from last lecture:

# Differential Input Low Gain Op Amps

Will Next Show That :

- Differential input op amps can be readily obtained from single-ended op amps
- Performance characteristics of differential op amps can be directly determined from those of the single-ended counterparts

Review from last lecture:

## Counterpart Networks

Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p-channel devices, replacing all p-channel devices with n-channel devices, replacing  $V_{SS}$  biases with  $V_{DD}$  biases, and replacing all  $V_{DD}$  biases with  $V_{SS}$  biases.

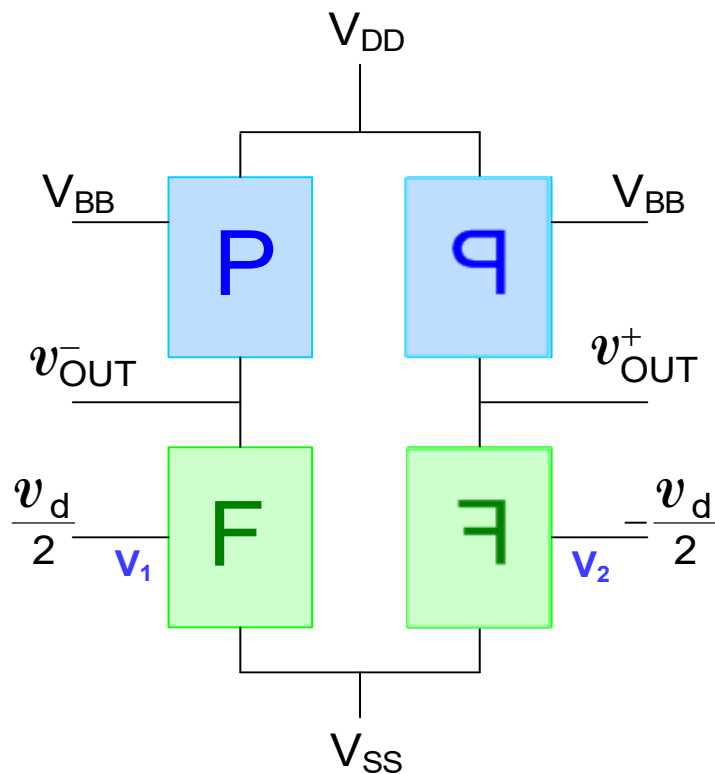
Review from last lecture:

## Counterpart Networks

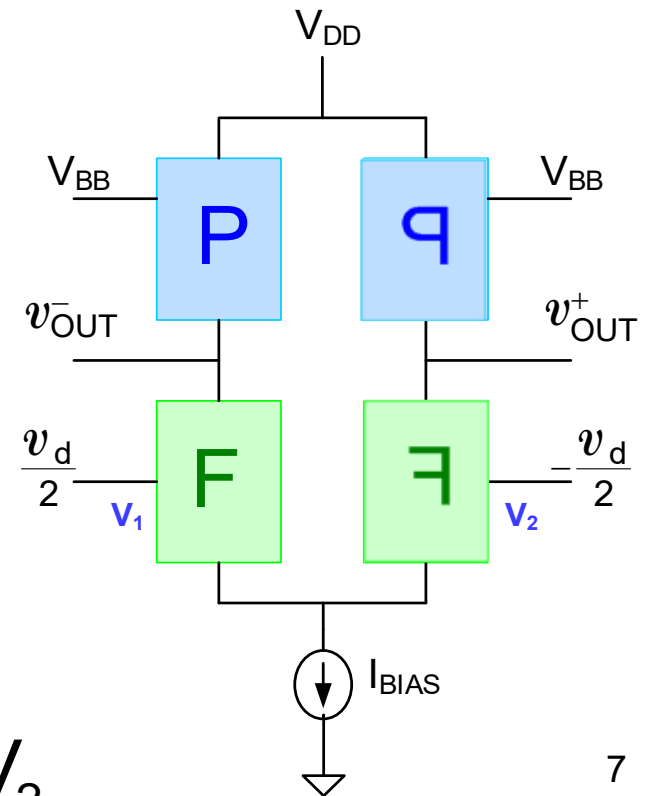
Theorem: The parametric expressions for all small-signal characteristics, such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.

# Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Theorem: If  $F$  is any network with a single input and  $P$  is its counterpart network, then the following circuits are fully differential circuits --- “op amps”.

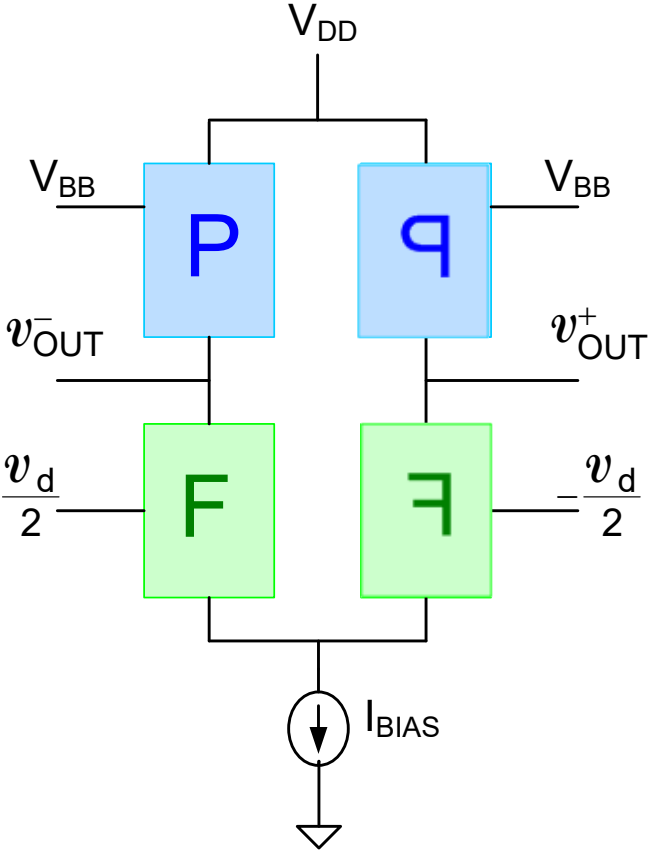


$$V_d = V_1 - V_2$$

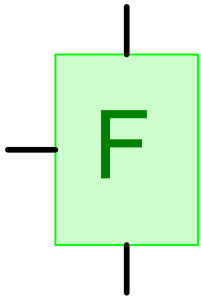


# Synthesis of fully-differential op amps from symmetric networks and counterpart networks

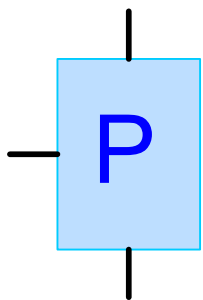
## Terminology



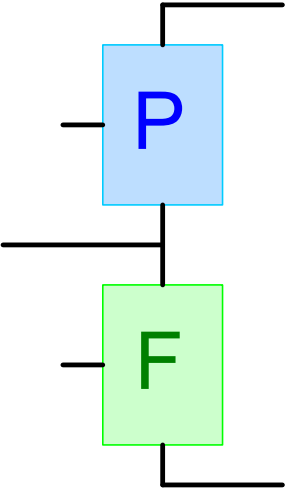
$$V_d = V_1 - V_2$$



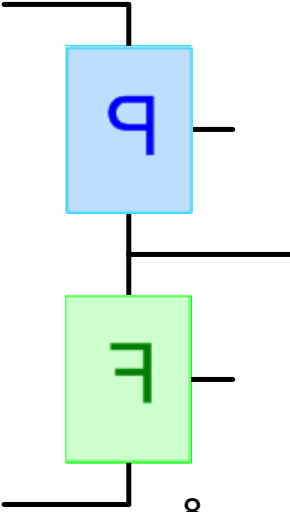
Quarter Circuit



Counterpart Circuit



Half Circuit



Symmetric Half Circuit

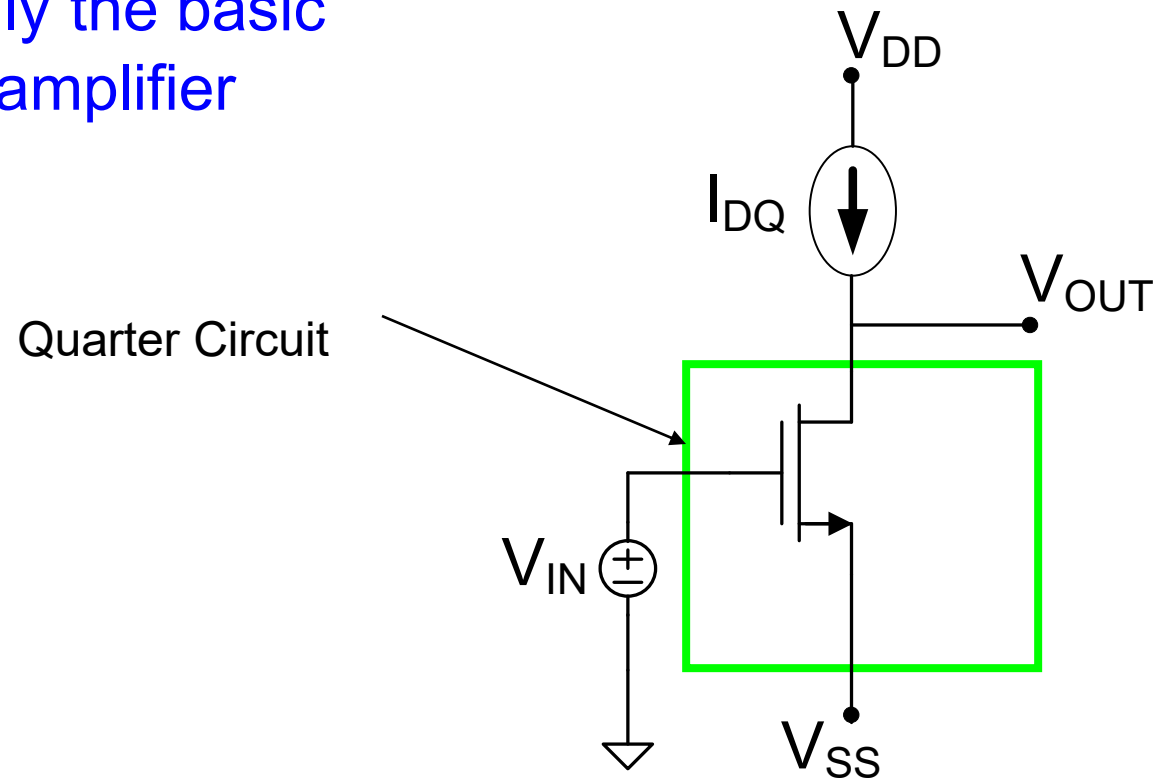
Review from last lecture:



Review from last lecture:

# Applications of Quarter-Circuit Concept to Op Amp Design

consider initially the basic single-ended amplifier

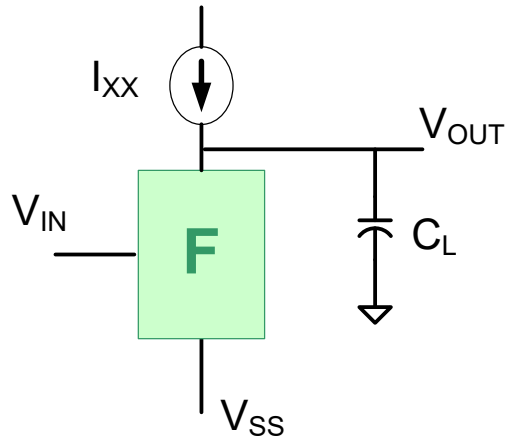


Review from last lecture:

# Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit



$$A_{voqc} = -\frac{G_M}{G}$$

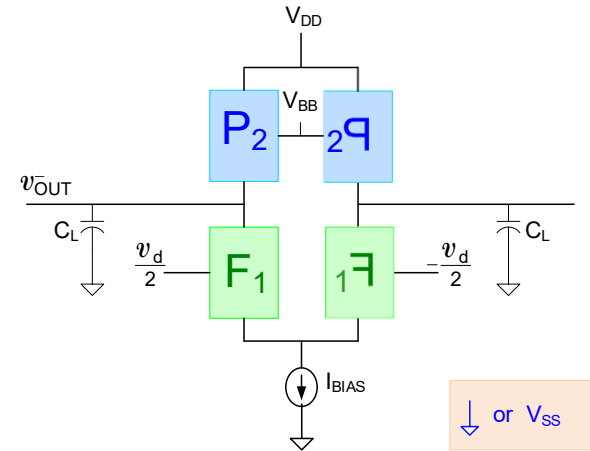
$$BW = \frac{G}{C_L} \quad GB = \frac{G_M}{C_L} \quad \longleftrightarrow$$

Note: Factor of 4 reduction of gain if  $G_1=G_2$  (this often occurs)

Note: Factor of 2 increase of BW if  $G_1=G_2$  (this often occurs)

Note: Factor of 2 reduction of GB if  $G_1=G_2$  (this often occurs)

Small signal differential amplifier



$$A_V = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

$$A_{V0} = \frac{v_{OUT}^-}{v_d} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

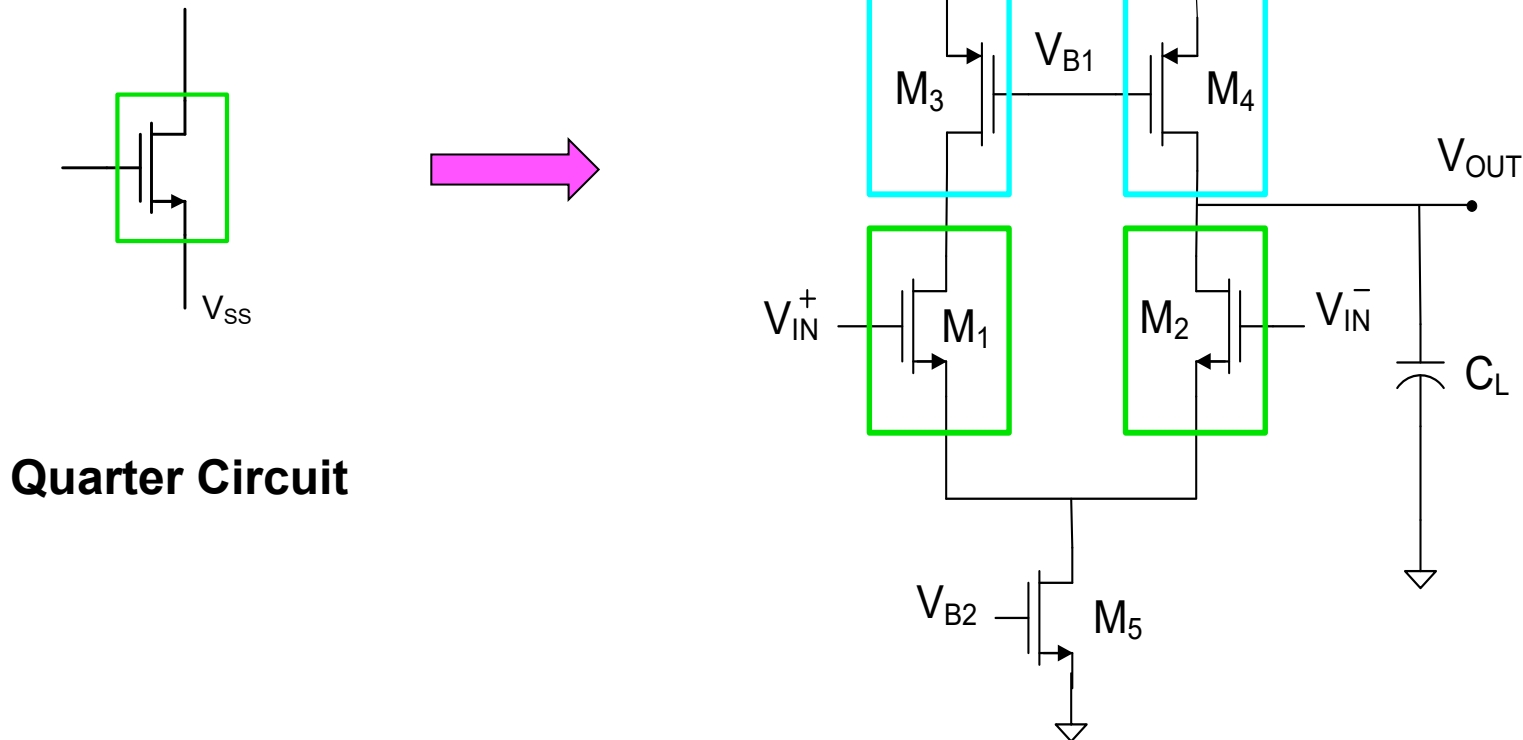
Remember this is applicable to ANY quarter circuit !

## Review from last lecture:

# Single-stage low-gain differential op amp

-- The “differential” gain --

Single-Ended Output : Differential Input Gain

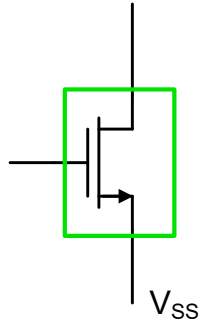


Have synthesized fully differential op amp from quarter circuit !

**Termed the 5T Op Amp**

# Single-stage low-gain differential op amp

-- The "differential" gain --



Quarter Circuit

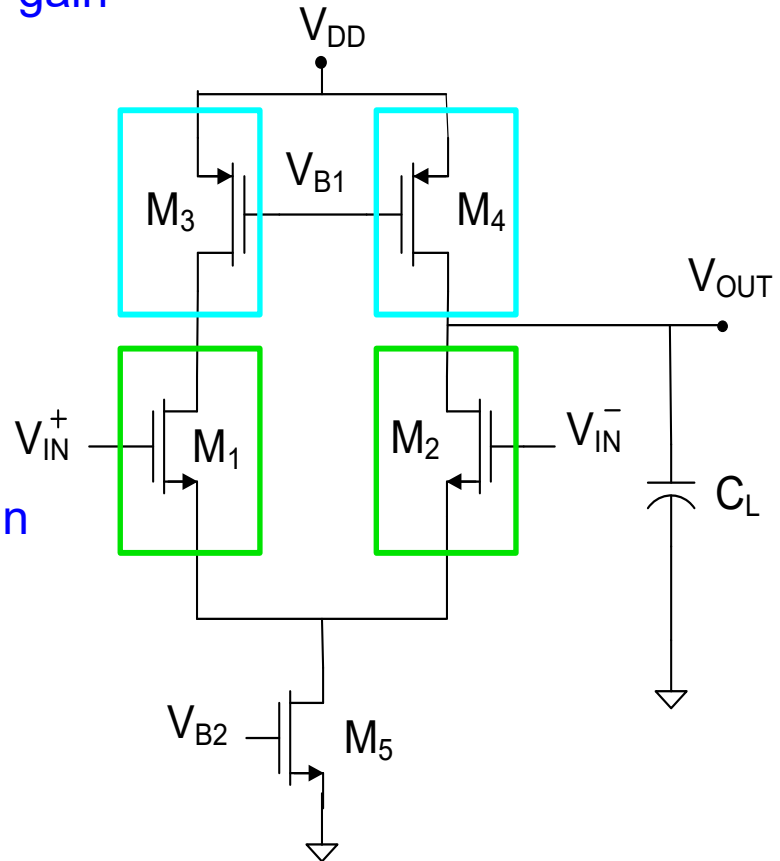
Single-Ended Output : Differential Input Gain

$$A(s) = \frac{v_{OUT}}{v_d} = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$A_{V0} = \frac{-g_{m1}}{2(g_{o1} + g_{o3})}$$

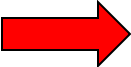
$$BW = \frac{g_{o1} + g_{o3}}{C_L}$$

$$GB = \frac{g_{m1}}{2C_L}$$



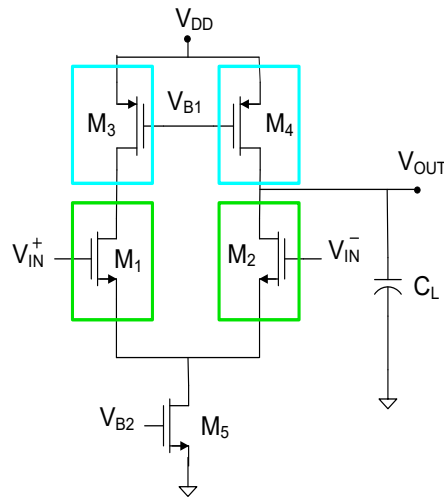
**Circuit is Very Sensitive to  $V_{B1}$  and  $V_{B2}$  !!**

- Have obtained analysis of fully differential op amp directly from quarter circuit !
- Still need to determine what happens if input is not differential !
- Have almost obtained op amp characteristics by inspection from quarter circuit !!

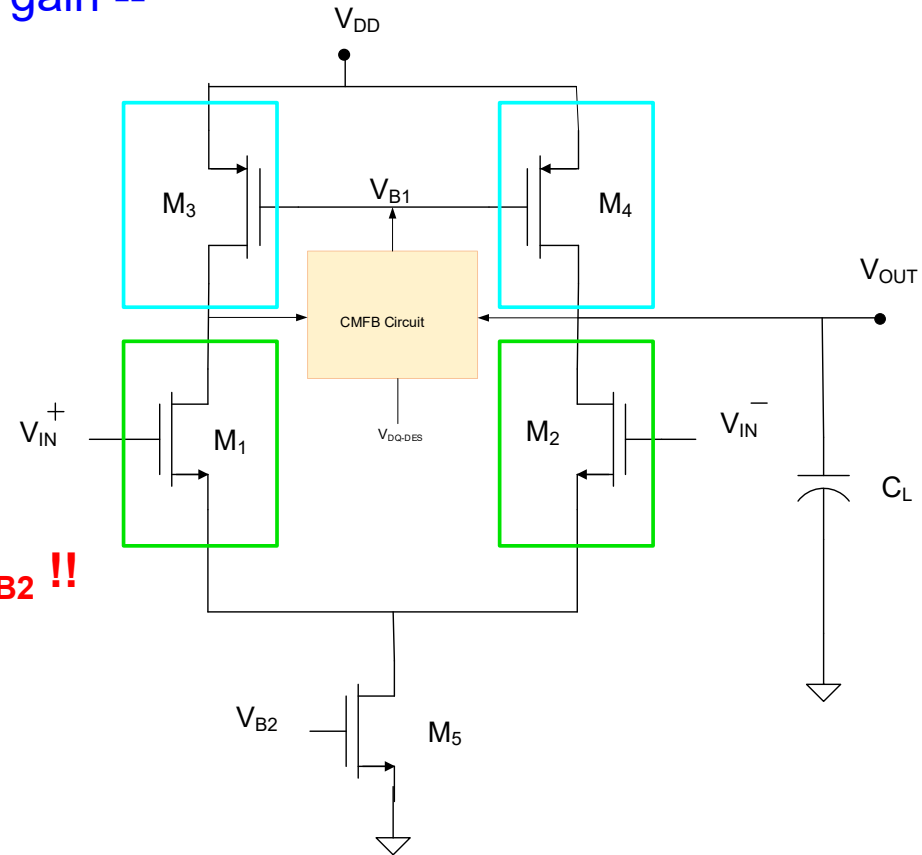
- Fully Differential Single-Stage Amplifier
  - General Differential Analysis
  - 5T Op Amp from simple quarter circuit
  -  – Biasing with CMFB circuit
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  - Overall Transfer Characteristics
- Design of 5T Op Amp
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# Single-stage low-gain differential op amp

-- The “differential” gain --



**Need CMFB circuit to establish  $V_{B1}$  or  $V_{B2}$  !!**

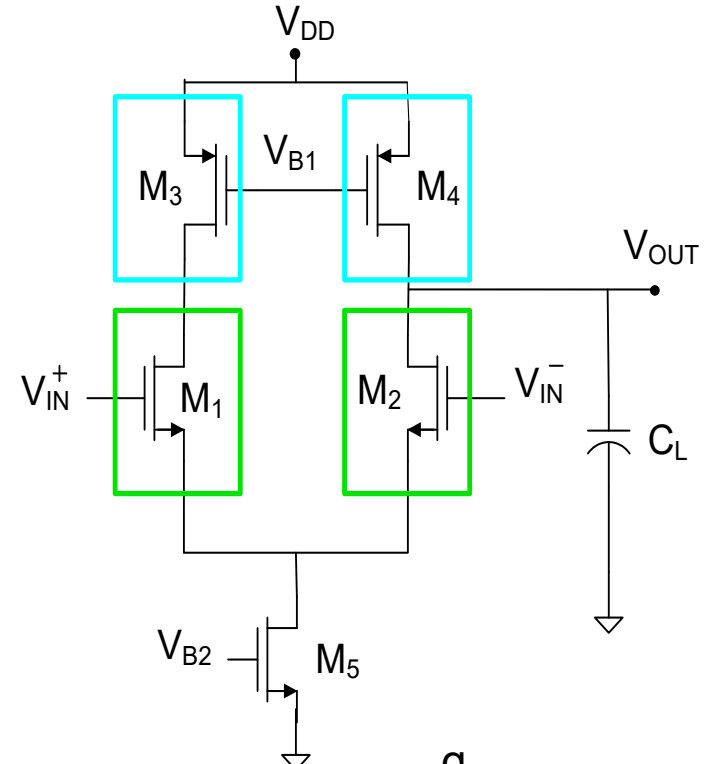
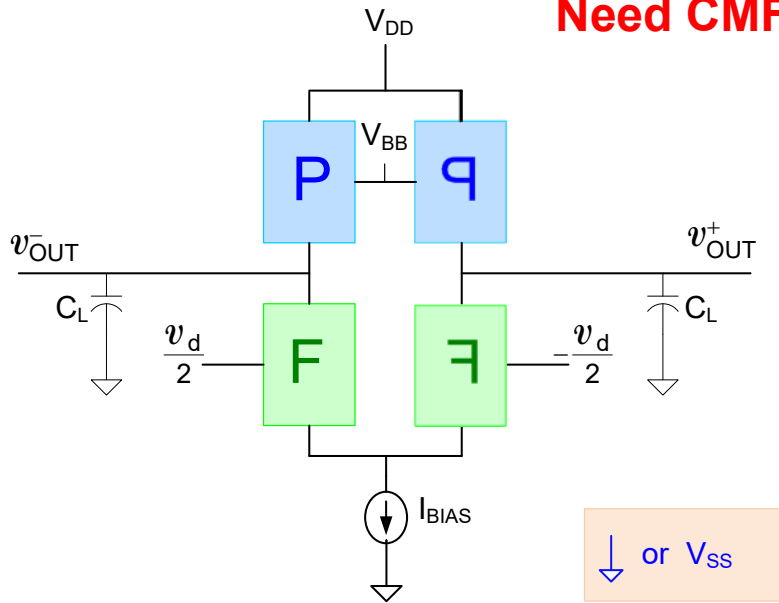


- CMFB circuit determines average value of the drain voltages
- Compares the average to the desired quiescent drain voltages
- Established a feedback signal  $V_{B1}$  to set the right Q-point
- Shown for  $V_{B1}$  but could alternately be applied to  $V_{B2}$

Details about CMFB circuits will be discussed later

# Single-stage low-gain differential op amp

-- The "differential" gain --  
**Need CMFB circuit**



$$A(s) = \frac{v_{OUT}^-}{v_d} = \frac{-\frac{G_M}{2}}{sC_L + G_1 + G_2}$$

$$A_{V0} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

$$BW = \frac{G_1 + G_2}{C_L}$$


$$GB = \frac{G_{M1}}{2C_L}$$

$$A(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$GB = \frac{g_{m1}}{2C_L}$$

$$A_o = \frac{\frac{g_{m1}}{2}}{g_{o1} + g_{o3}}$$

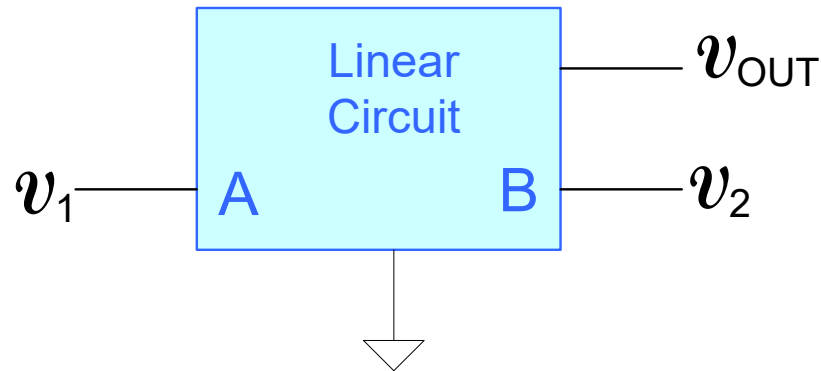
Have obtained differential gain of 5T Op Amp by inspection from quarter circuit

- Fully Differential Single-Stage Amplifier
  - General Differential Analysis
  - 5T Op Amp from simple quarter circuit
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# Common-Mode and Differential-Mode Analysis

Consider an output voltage for any linear circuit with two inputs  
(i.e. need not be symmetric)



By superposition

$$v_{OUT} = A_1 v_1 + A_2 v_2$$

where  $A_1$  and  $A_2$  are the gains (transfer functions) from inputs 1 and 2 to the output respectively

Define the common-mode and difference-mode inputs by

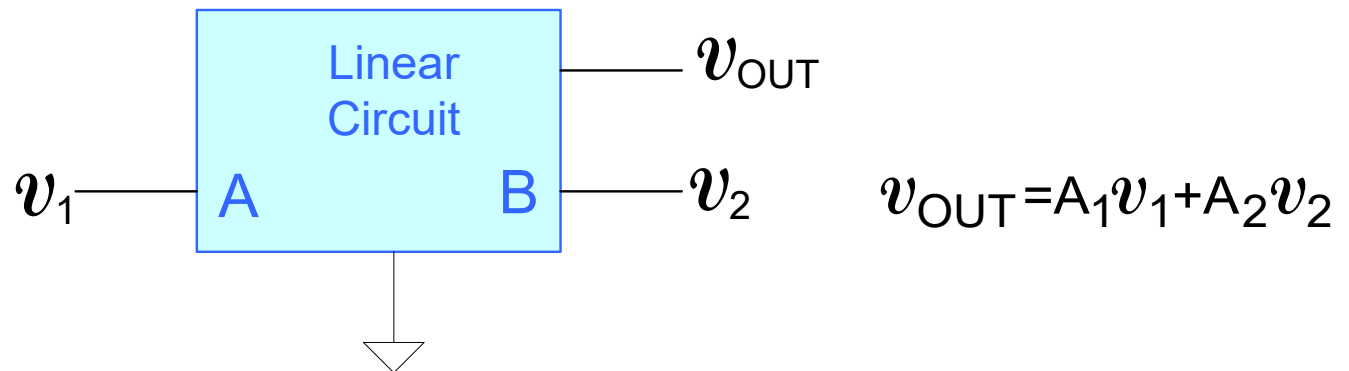
$$v_c = \frac{v_1 + v_2}{2} \qquad v_d = v_1 - v_2$$

These two equations can be solved for  $v_1$  and  $v_2$  to obtain

$$v_1 = v_c + \frac{v_d}{2} \qquad v_2 = v_c - \frac{v_d}{2}$$

# Common-Mode and Differential-Mode Analysis

Consider an output voltage for any linear circuit with two inputs



Substituting into the expression for  $v_{OUT}$ , we obtain

$$v_{OUT} = A_1 \left( v_c + \frac{v_d}{2} \right) + A_2 \left( v_c - \frac{v_d}{2} \right)$$

Rearranging terms we obtain

$$v_{OUT} = v_c (A_1 + A_2) + v_d \left( \frac{A_1 - A_2}{2} \right)$$

If we define  $A_c$  and  $A_d$  by

$$A_c = A_1 + A_2 \qquad A_d = \frac{A_1 - A_2}{2}$$

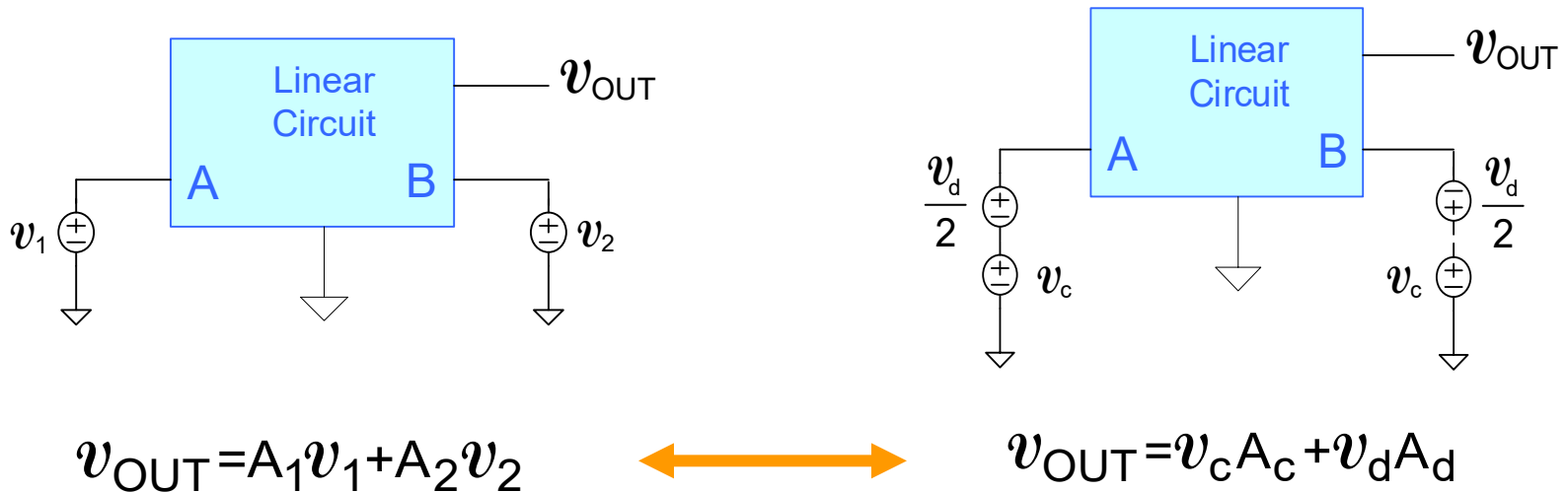
Can express  $v_{OUT}$  as

$$v_{OUT} = v_c A_c + v_d A_d$$

# Common-Mode and Differential-Mode Analysis

Depiction of single-ended inputs and common/difference mode inputs

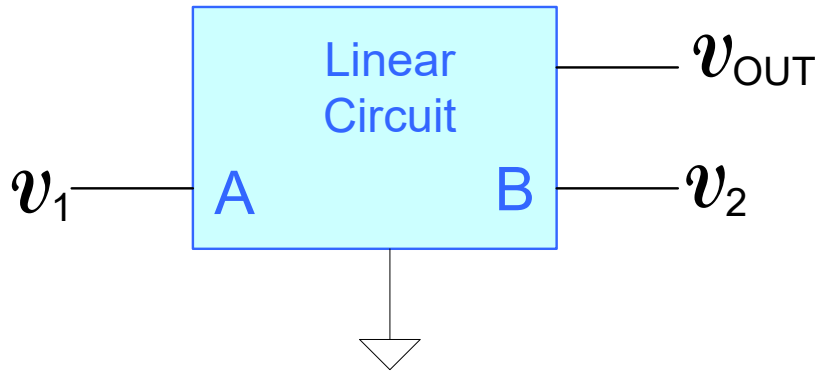
## Alternate Equivalent Representations



- Applicable to any linear circuit with two inputs and a single output
- Op amps often have symmetry and this symmetry further simplifies analysis

# Common-Mode and Differential-Mode Analysis

Consider any output voltage for any linear circuit with two inputs



$$A_c = A_1 + A_2$$

$$A_d = \frac{A_1 - A_2}{2}$$

$$v_{OUT} = A_1 v_1 + A_2 v_2$$

$$v_{OUT} = v_c A_c + v_d A_d$$

$$v_{OUT} = v_c (A_1 + A_2) + v_d \left( \frac{A_1 - A_2}{2} \right)$$

Implication: Can solve any linear two-input circuit by applying superposition with  $v_1$  and  $v_2$  as inputs or with  $v_c$  and  $v_d$  as inputs. This can be summarized in the following theorem:

Theorem 1: The output for any linear network can be expressed equivalently as  $v_{OUT} = A_1 v_1 + A_2 v_2$  or as  $v_{OUT} = v_c A_c + v_d A_d$

Superposition can be applied to either  $v_1$  and  $v_2$  to obtain  $A_1$  and  $A_2$  or to  $v_c$  and  $v_d$  to obtain  $A_c$  and  $A_d$

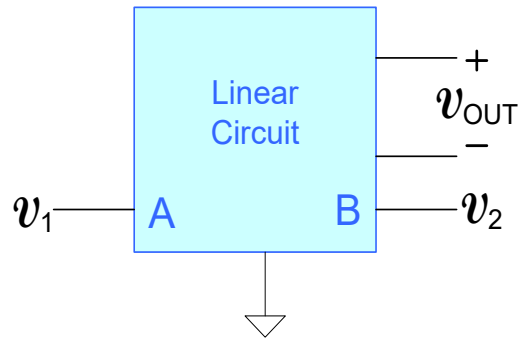
Observation: In a circuit with  $A_2 = -A_1$ ,  $A_c = 0$  we obtain

$$v_{OUT} = v_d A_d$$

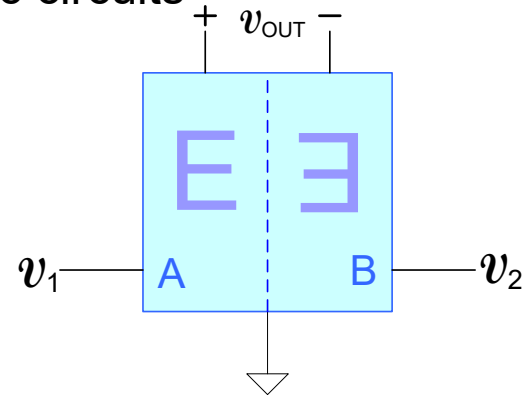
Analysis of op amps up to this point have assumed differential excitation

# Common-Mode and Differential-Mode Analysis

Extension to differential outputs and symmetric circuits



**Differential Output**



**Symmetric Circuit with Symmetric Differential Output**

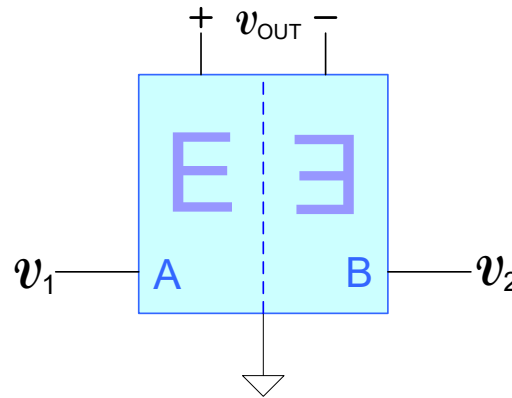
Observation: In a symmetric circuit with a symmetric differential output,  $A_C=0$  so can be shown that  $v_{OUT}=v_d A_d$  This is summarized in the theorem:

Theorem 2: The symmetric differential output voltage for any symmetric linear network excited at symmetric nodes can be expressed as

$$v_{OUT}=A_d v_d$$

where  $A_d$  is the differential voltage gain and the voltage  $v_d = v_1 - v_2$

# Symmetric Circuit with Symmetric Differential Output



Theorem 2: The symmetric differential output voltage for any symmetric linear network excited at symmetric nodes can be expressed as

$$v_{OUT} = A_d v_d$$

where  $A_d$  is the differential voltage gain and the voltage  $v_d = v_1 - v_2$

# Common-Mode and Differential-Mode Analysis

## Proof of Theorem 2 for Symmetric Circuit with Symmetric Differential Output:

By superposition, the single-ended outputs can be expressed as

$$v_{OUT+} = T_{0PA}v_1 + T_{0PB}v_2$$

$$v_{OUT-} = T_{0NA}v_1 + T_{0NB}v_2$$

where  $T_{0PA}$ ,  $T_{0PB}$ ,  $T_{0NA}$  and  $T_{0NB}$  are the transfer functions from the A and B inputs to the single-ended + and - outputs

taking the difference of these two equations we obtain

$$v_{OUT} = v_{OUT+} - v_{OUT-} = (T_{0PA} - T_{0NA})v_1 + (T_{0PB} - T_{0NB})v_2$$

by symmetry, we have

$$T_{0PA} = T_{0NB} \text{ and } T_{0NA} = T_{0PB}$$

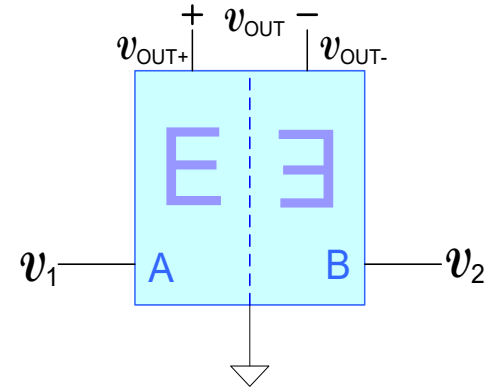
thus can express  $v_{OUT}$  as

$$v_{OUT} = (T_{0PA} - T_{0NA})(v_1 - v_2)$$

or as

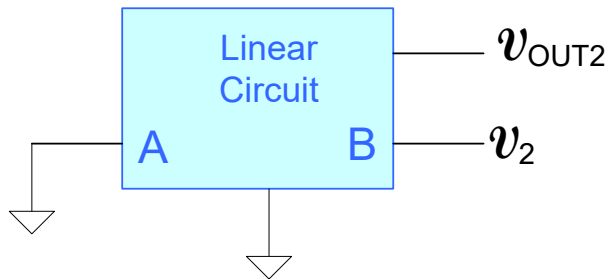
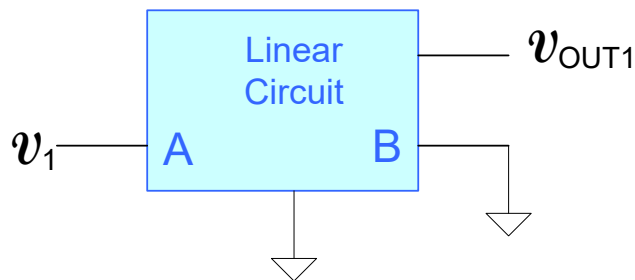
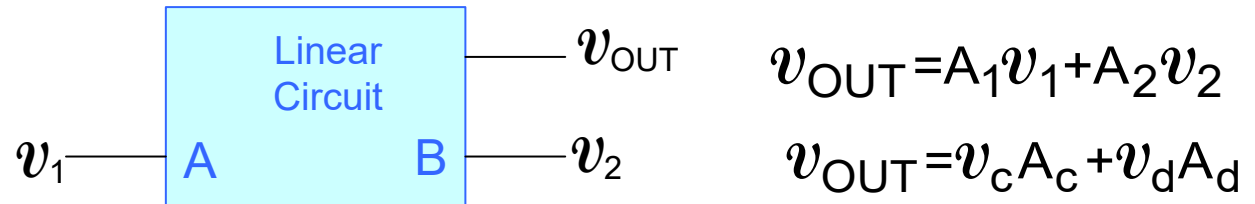
$$v_{OUT} = A_d v_d$$

where  $A_d = T_{0PA} - T_{0NA}$  and where  $v_d = v_1 - v_2$

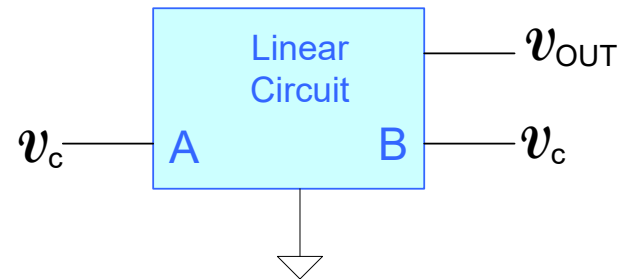
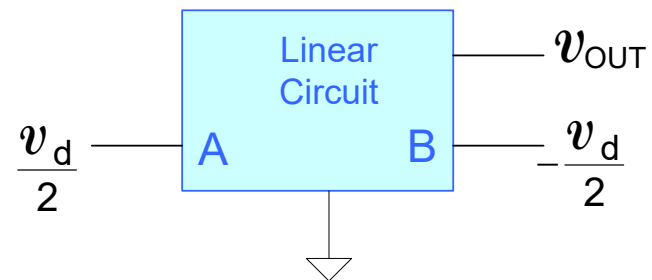


# Common-Mode and Differential-Mode Analysis

Consider any output voltage for any linear circuit with two inputs



Single-Ended Superposition

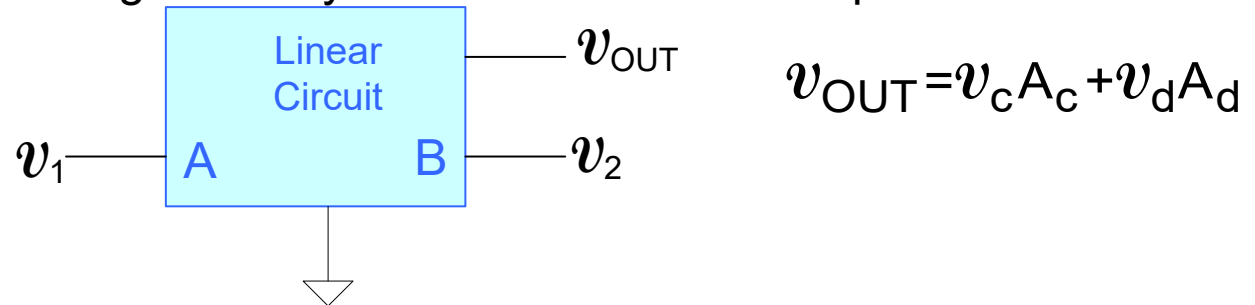


Difference-Mode/Common-Mode Superposition

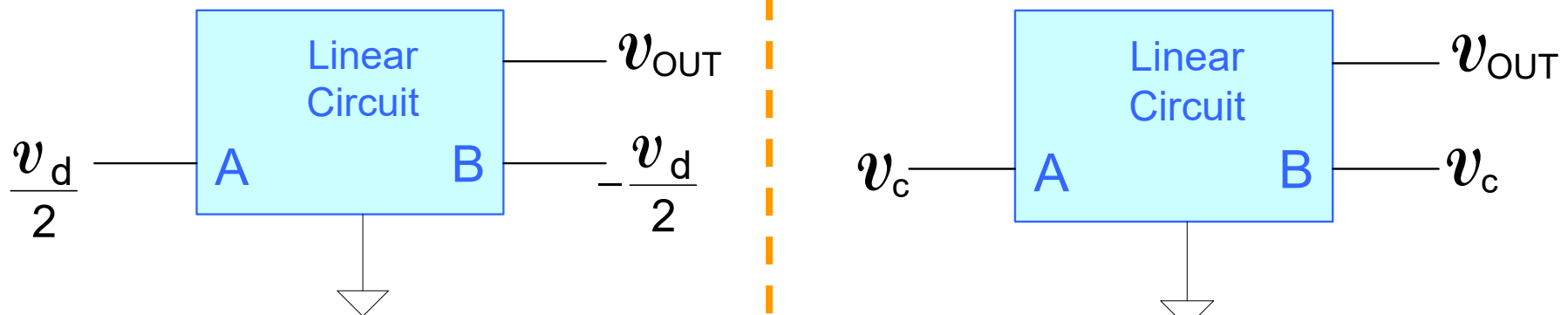


# Common-Mode and Differential-Mode Analysis

Consider an output voltage for any linear circuit with two inputs



- **Difference-Mode/Common-Mode Superposition is almost exclusively used for characterizing Amplifiers that are designed to have a large differential gain and a small common-mode gain**
- **Analysis to this point has been focused only on the circuit on the left**

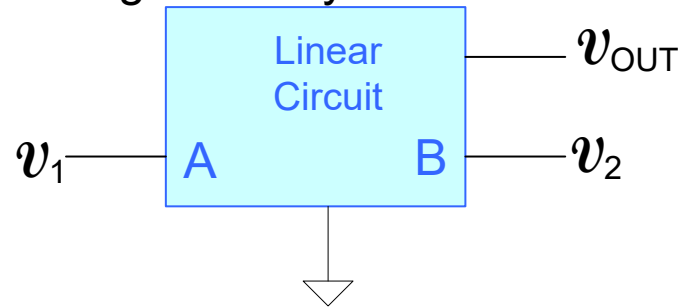


Note: Previous analysis was correct, just did not address whether the circuit had any common mode gain.

**Will now get the total output of an amplifier circuit !**

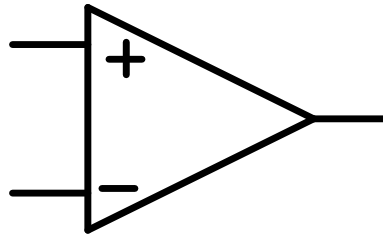
# Common-Mode and Differential-Mode Analysis

Consider an output voltage for any linear circuit with two inputs



$$v_{\text{OUT}} = v_c A_c + v_d A_d$$

Does Conventional Wisdom Address the Common Mode Gain Issue?



# Does Conventional Wisdom Address the Common Mode Gain Issue?

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JPEG Image  
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Resolution: 2144 x 2832 pixels

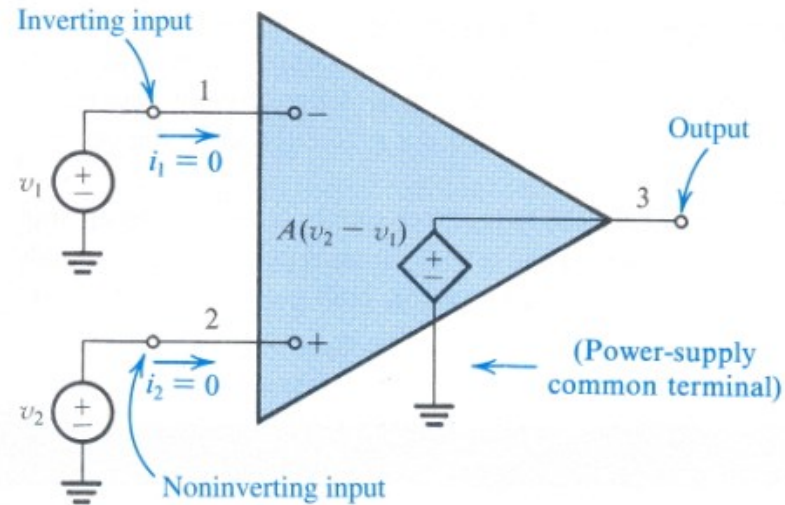


FIGURE 2.3 Equivalent circuit of the ideal op amp.



Yes – Common-Mode Gain was Addressed

# Does Conventional Wisdom Address the Common Mode Gain Issue?

Page.jpg  
JPEG Image  
1.46 MB  
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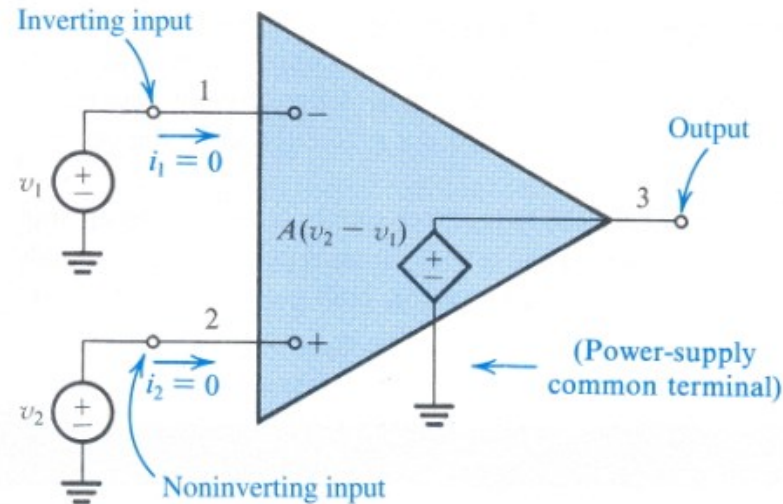


FIGURE 2.3 Equivalent circuit of the ideal op amp.

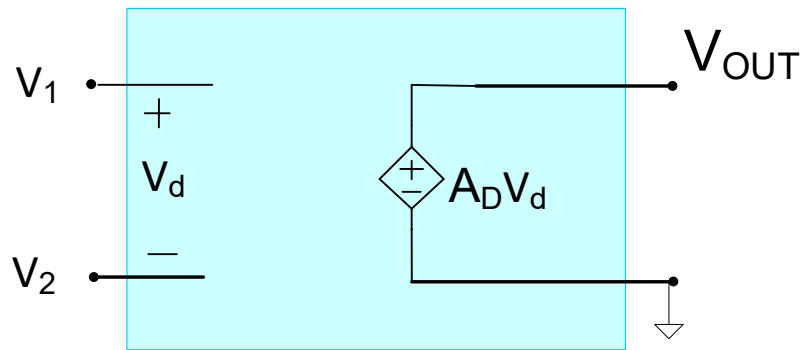
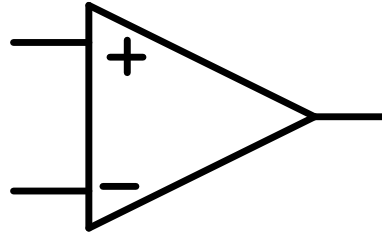
TABLE 2.1 Characteristics of the Ideal Op Amp

1. Infinite input impedance
2. Zero output impedance
- 3. Zero common-mode gain or, equivalently, infinite common-mode rejection
4. Infinite open-loop gain  $A$
5. Infinite bandwidth

Yes – Common-Mode Gain was Addressed

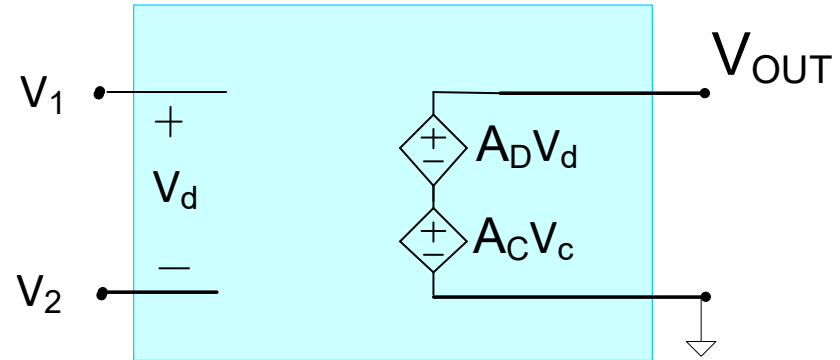
# How is Common-Mode Gain Modeled?

If Op Amp is a Voltage Amplifier with infinite input impedance, zero output impedance, and one terminal of the output is grounded




Ideal Differential Voltage Amplifier

$$V_d = V_1 - V_2$$

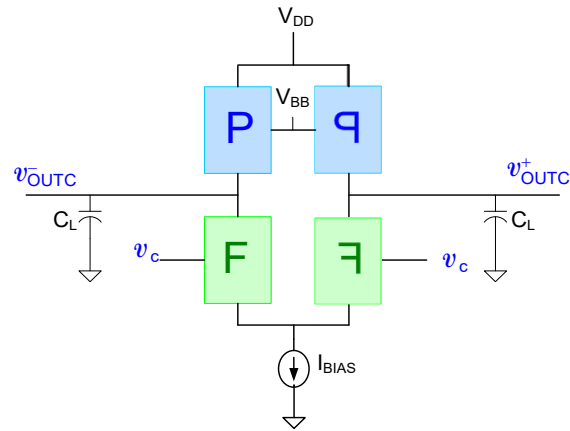


Ideal Voltage Amplifier

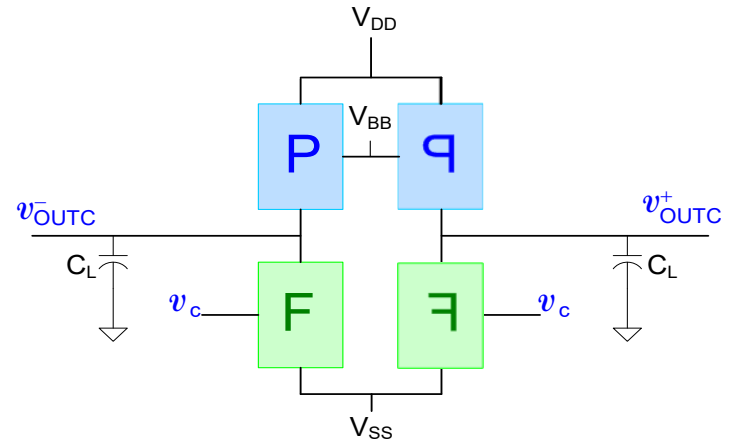
$$V_d = V_1 - V_2 \quad V_c = \frac{V_1 + V_2}{2}$$

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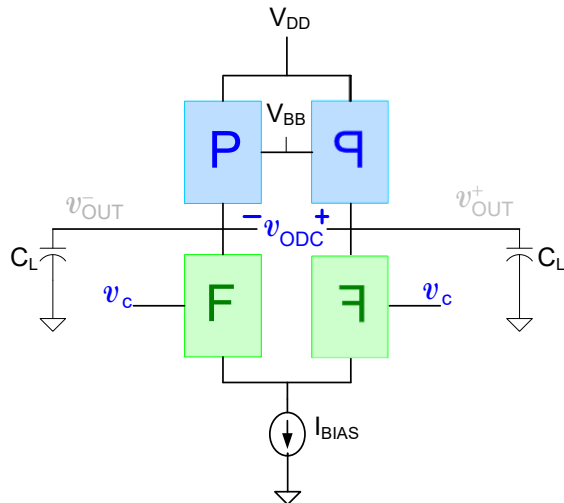
# Performance with Common-Mode Input



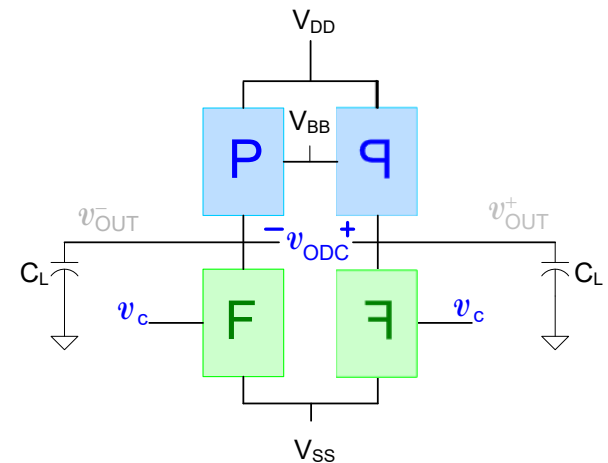
Single-Ended Outputs  
Tail-Current Bias



Single-Ended Outputs  
Tail-Voltage Bias



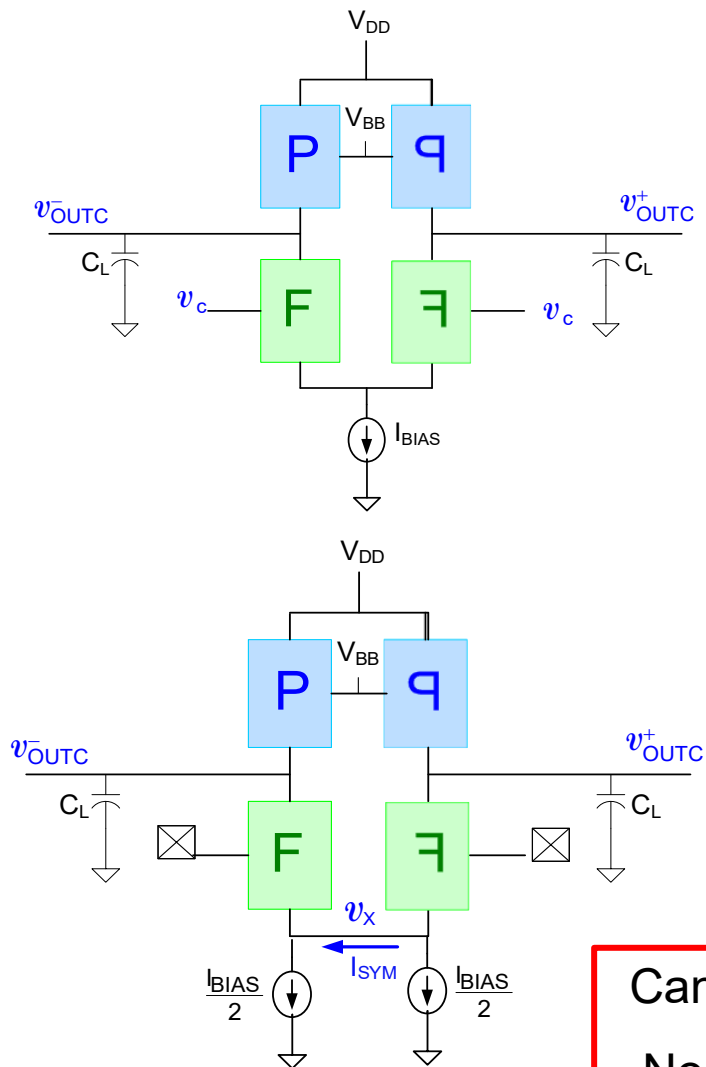
Differential Output  
Tail Current Bias



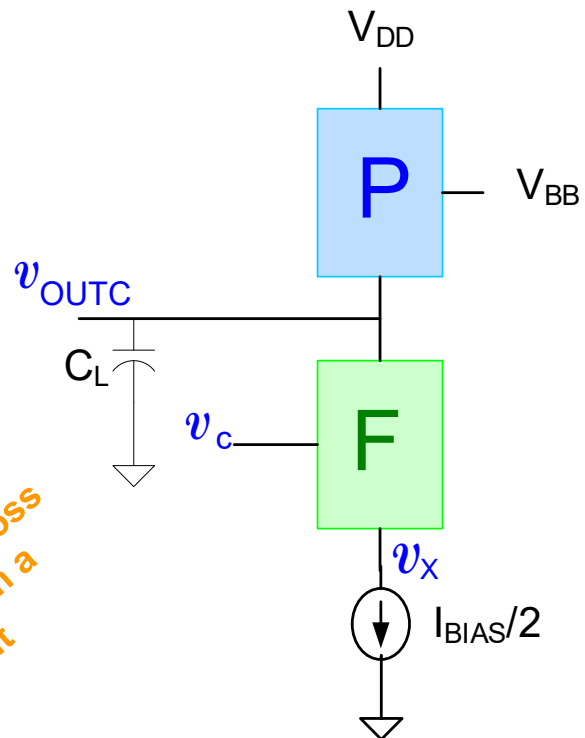
Differential Output  
Tail Voltage Bias

# Performance with Common-Mode Input

Consider tail-current bias amplifier



*No current flows across axis of symmetry in a symmetric circuit*



Common-Mode Half-Circuit

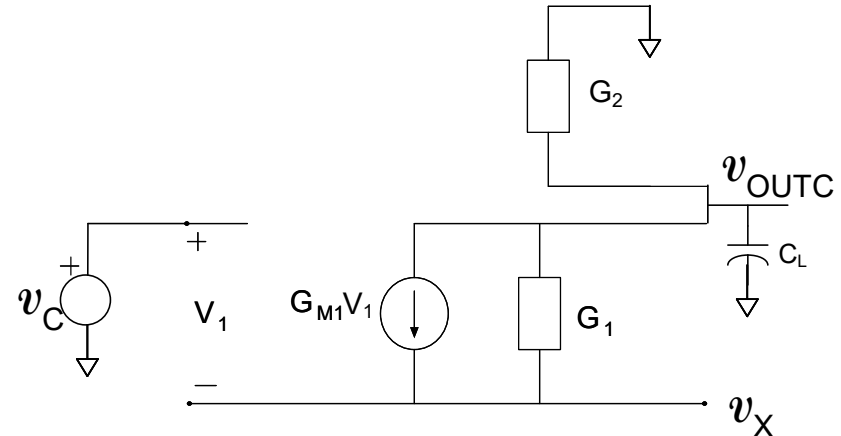
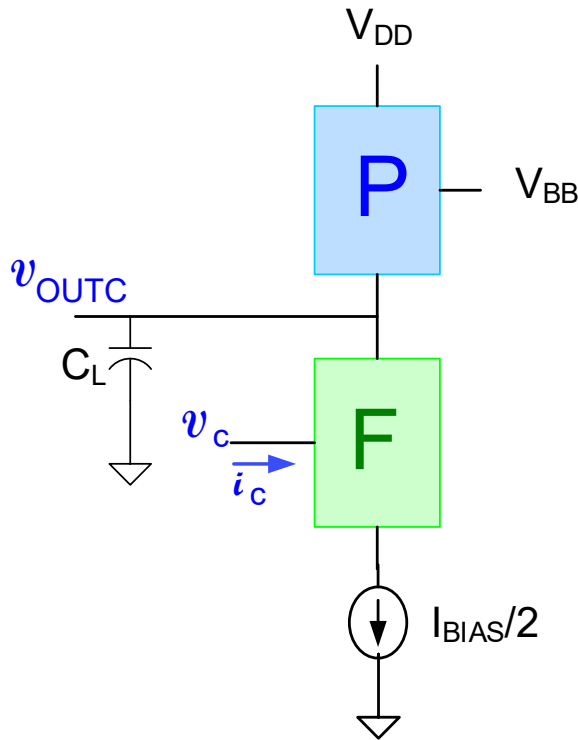
Can we assume  $v_x=0$  since it is on axis of symmetry?

No! Excitation is not differential !



# Performance with Common-Mode Input

Consider tail-current bias amplifier with  $i_c=0$



Common-Mode Half-Circuit  
(small signal: linear)

$$\left. \begin{aligned} v_{OUTC}(sC+G_1+G_2)+G_{M1}v_1 &= G_1v_x \\ v_c &= v_1+v_x \\ v_xG_1 - G_{M1}v_1 &= v_{OUTC}G_1 \end{aligned} \right\}$$

Solving, we obtain

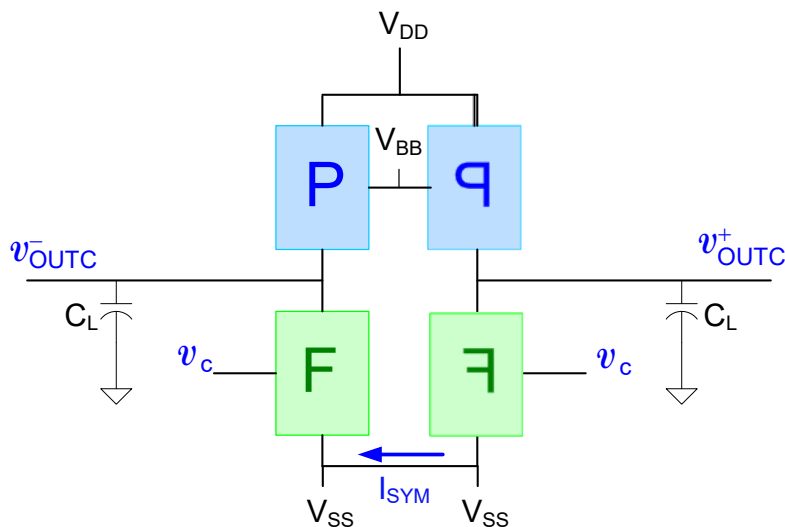
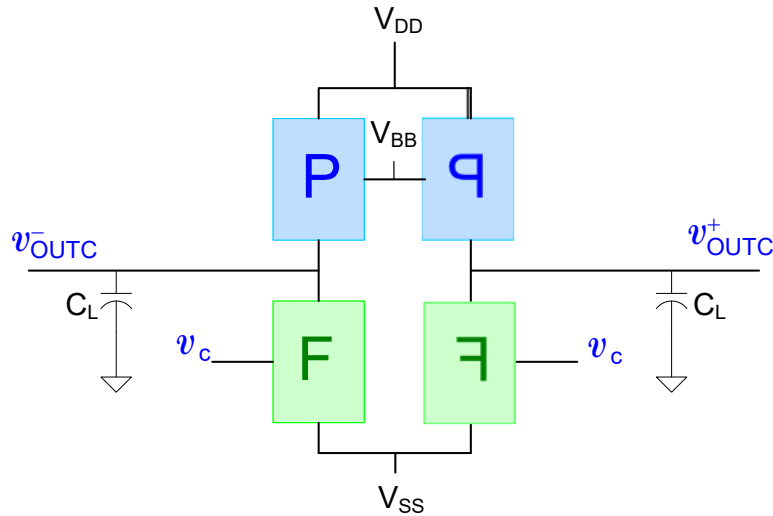
$$v_{OUTC}=0 \quad \text{thus } A_C=0$$

Common-Mode Half-Circuit  
(large signal: nonlinear)

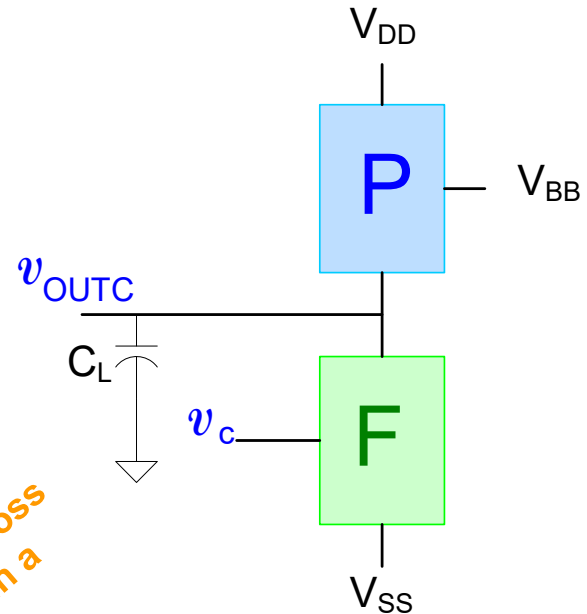
(Note: Have assumed an ideal tail current source in this analysis  $A_C$  will be small but may not vanish if tail current source is not ideal. Analysis with nonideal current source is simple)

# Performance with Common-Mode Input

Consider tail-voltage bias amplifier with  $i_c=0$



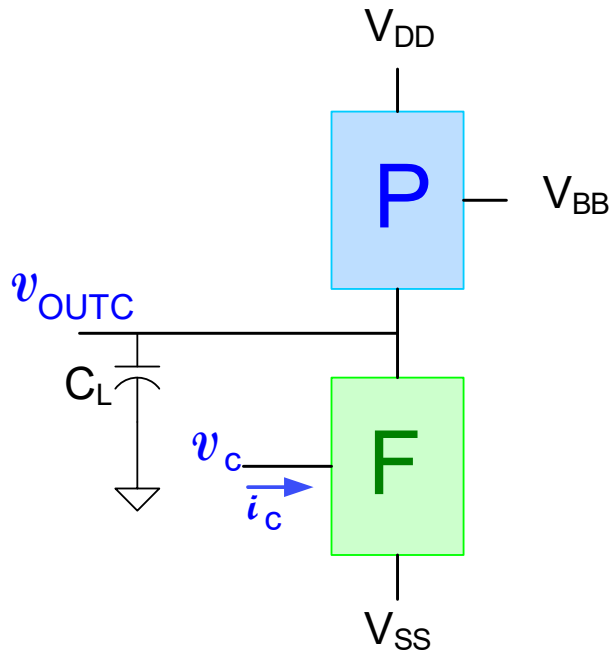
No current flows across axis of symmetry in a symmetric circuit



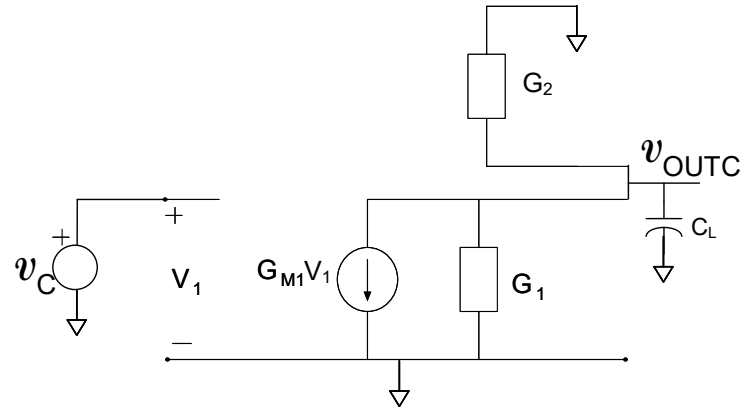
Common-Mode Half-Circuit

# Performance with Common-Mode Input

Consider tail-voltage bias amplifier with  $i_c=0$



Common-Mode Half-Circuit  
(large signal: nonlinear)



Common-Mode Half-Circuit  
(small signal: linear)

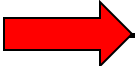
$$\left. \begin{aligned} v_{OUTC}(sC+G_1+G_2)+G_{M1}v_1 &= 0 \\ v_C &= v_1 \end{aligned} \right\}$$

Solving, we obtain

$$\frac{v_{OUTC}}{v_C} = A_C = \frac{-G_{M1}}{sC+G_1+G_2}$$

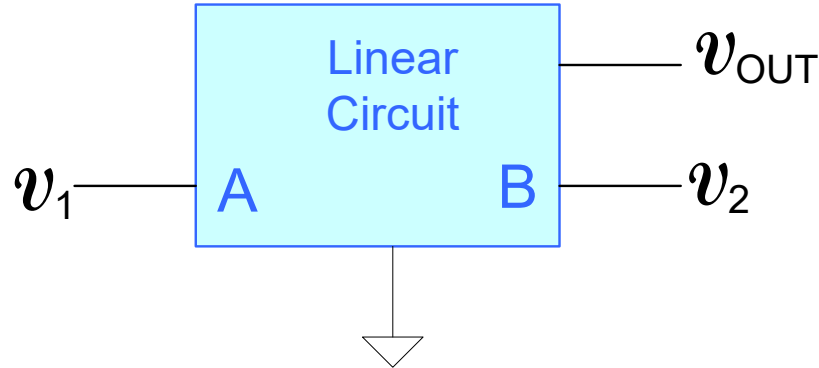
This circuit has a rather large common-mode gain and will not reject common-mode signals

- Not a very good differential amplifier
- But of no concern in applications where  $v_c=0$

- Fully Differential Single-Stage Amplifier
  - General Differential Analysis
  - 5T Op Amp from simple quarter circuit
  - Biasing with CMFB circuit
  - Common-mode and differential-mode analysis
  - Common Mode Gain
-  Overall Transfer Characteristics
- Design of 5T Op Amp
- Slew Rate

# Overall Small-Signal Analysis

As stated earlier, with common-mode gain and difference-mode gains available



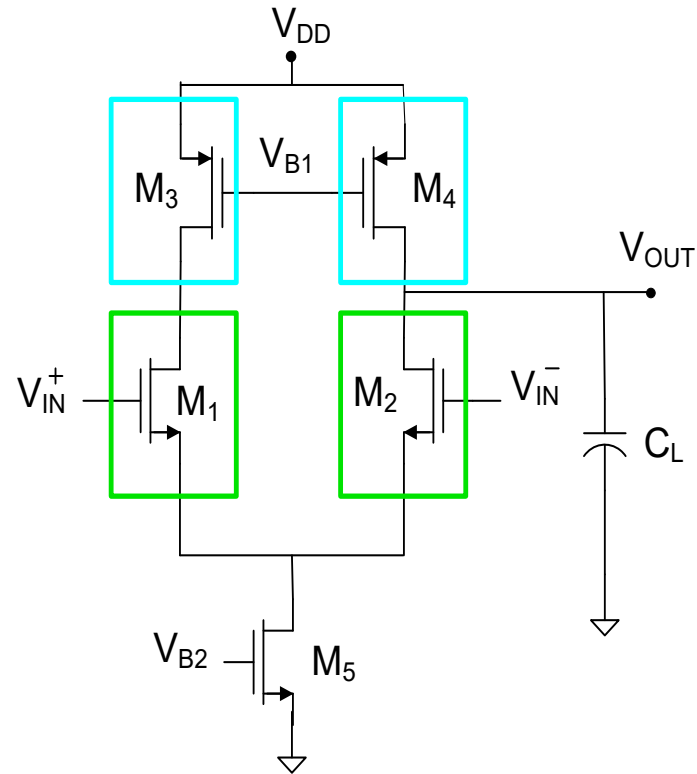
$$v_{OUT} = v_c A_c + v_d A_d$$

- Fully Differential Single-Stage Amplifier
  - General Differential Analysis
  - 5T Op Amp from simple quarter circuit
  - Biasing with CMFB circuit
  - Common-mode and differential-mode analysis
  - Common Mode Gain
  - Overall Transfer Characteristics

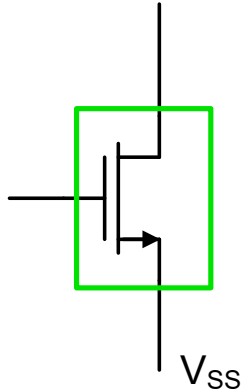
 Design of 5T Op Amp

- Slew Rate

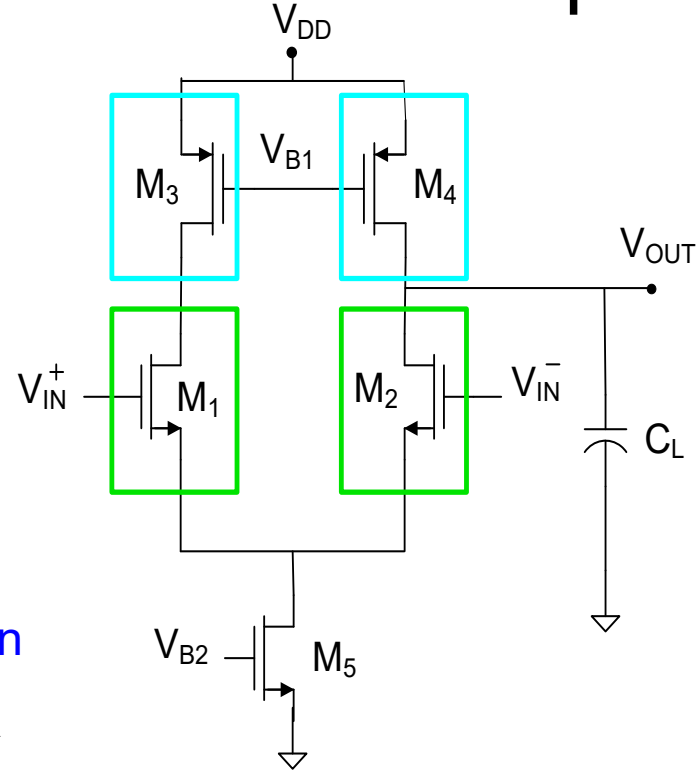
# Design of 5T op amp



# Single-stage low-gain differential op amp



**Quarter Circuit**



Single-Ended Output : Differential Input Gain

$$A(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$A_o = \frac{\frac{g_{m1}}{2}}{g_{o1} + g_{o3}}$$

$$GB = \frac{g_{m1}}{2C_L}$$

**Need a CMFB circuit to establish  $V_{B1}$**

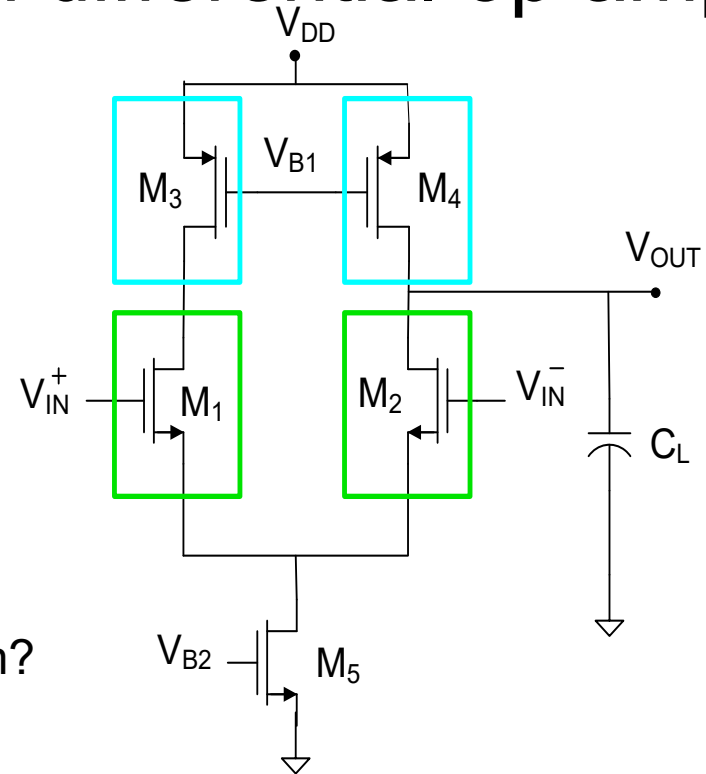


# Design of Basic Single-stage low-gain differential op amp

$$A(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$A_o = \frac{\frac{g_{m1}}{2}}{g_{o1} + g_{o3}}$$

$$GB = \frac{g_{m1}}{2C_L}$$



What are the number of degrees of freedom?  
(assume  $V_{DD}$ ,  $C_L$  fixed, Symmetry)

Natural Parameters (assuming symmetry):

$$\left\{ \frac{W_1}{L_1}, \frac{W_3}{L_3}, \frac{W_5}{L_5}, V_{B1}, V_{B2} \right\}$$

Constraints:  $I_{D5} \approx 2I_{D3}$

Net Degrees of Freedom: 4

**Need a CMFB circuit to establish  $V_{B1}$**

- Expressions for  $A_o$  and GB were obtained from quarter-circuit
- Expressions for  $A_o$  and GB in terms of natural parameters for quarter circuit were messy
- Can show that expressions for  $A_o$  and GB in terms of natural parameters for 5T amplifier are also messy

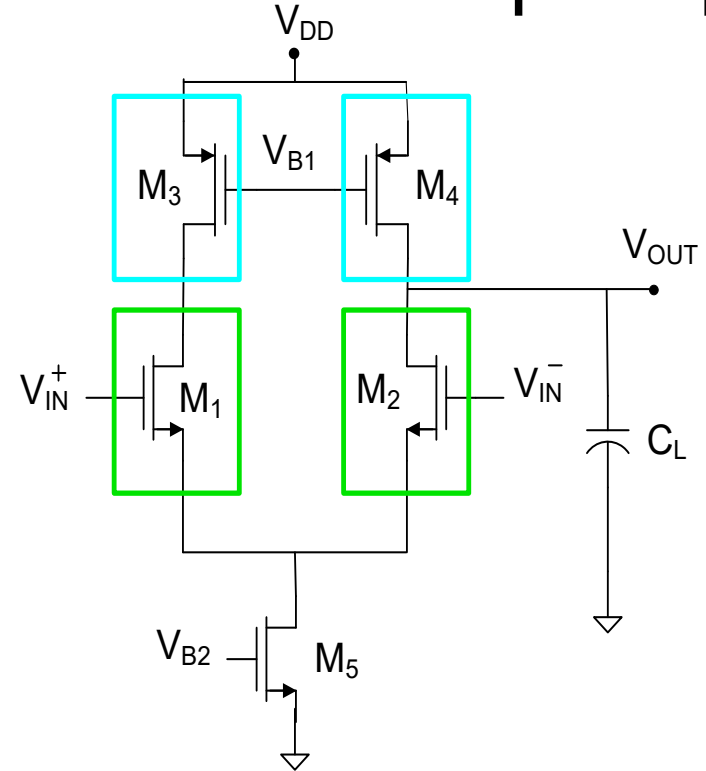
**Can a set of practical design parameters be identified?**

# Design of Basic Single-stage low-gain differential op amp

$$A(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$A_o = \frac{\frac{g_{m1}}{2}}{g_{o1} + g_{o3}}$$

$$GB = \frac{g_{m1}}{2C_L}$$



**Need a CMFB circuit to establish  $V_{B1}$**

What are the number of degrees of freedom?  
(assume  $V_{DD}$ ,  $C_L$  fixed, Symmetry)

Natural Parameters:

$$\left\{ \frac{W_1}{L_1}, \frac{W_3}{L_3}, \frac{W_5}{L_5}, V_{B1}, V_{B2} \right\}$$

Constraints:  $I_{D5} \approx 2I_{D3}$

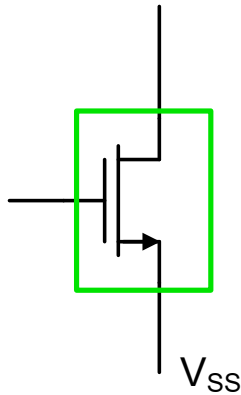
Net Degrees of Freedom: 4

Practical Parameters:

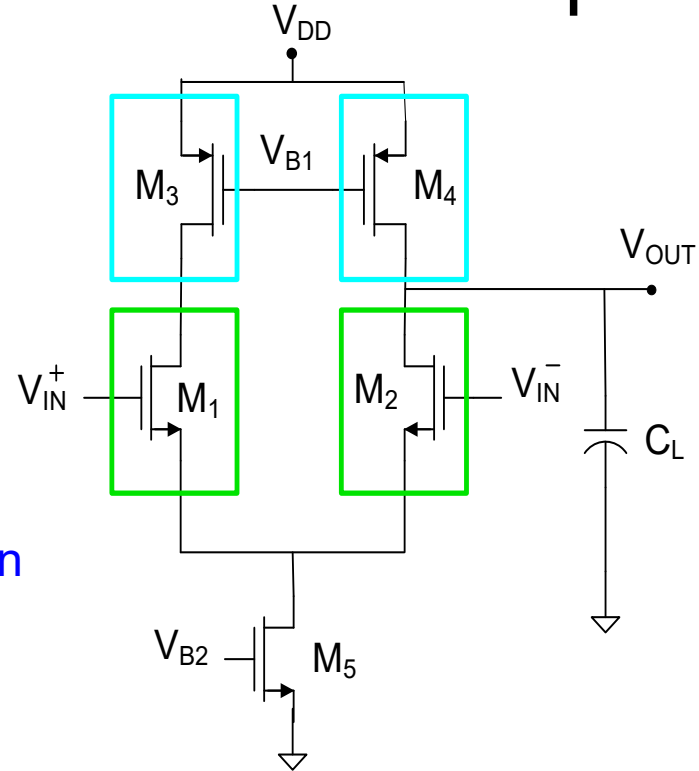
$$\{V_{EB1}, V_{EB3}, V_{EB5}, P\}$$

**Will now express performance characteristics in terms of Practical Parameters**

# Design of Basic Single-stage low-gain differential op amp



Quarter Circuit



Single-Ended Output : Differential Input Gain

$$A(s) = \frac{-\frac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$A_o = \frac{\frac{g_{m1}}{2}}{g_{o1} + g_{o3}}$$

$$GB = \frac{g_{m1}}{2C_L}$$

Practical Parameters:

$$\{V_{EB1}, V_{EB3}, V_{EB5}, P\}$$

$$A_o = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{1}{V_{EB1}} \right) \quad GB = \left( \frac{P}{V_{DD} C_L} \right) \cdot \left[ \frac{1}{2V_{EB1}} \right]$$

Have 4 degrees of freedom but only two practical variables impact  $A_o$  and  $GB$  so still have 2 DOF after meet  $A_o$  and  $GB$  requirements

**Need a CMFB circuit to establish  $V_{B1}$**



Stay Safe and Stay Healthy !

**End of Lecture 4**